Asymptotic soliton–like solutions of the Korteweg–de Vries equations with variable coefficients and singular perturbation

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The talk deals with the Korteweg-de Vries equation with variable coefficients (**vcKdV equation**) and a small parameter:

$$\varepsilon^n u_{xxx} = a(x,t,\varepsilon)u_t + b(x,t,\varepsilon)uu_x, \quad n \in \mathbf{N}, \tag{1}$$

where

$$a(x,t,\varepsilon) = \sum_{k=0}^{\infty} \varepsilon^k a_k(x,t), \quad b(x,t,\varepsilon) = \sum_{k=0}^{\infty} \varepsilon^k b_k(x,t), \tag{2}$$

with the functions $a_k(x,t)$, $b_k(x,t) \in C^\infty(\mathbf{R} \times [0;T])$, $k \geq 0$, $T > 0$, and

$$a_0(x,t)\, b_0(x,t) \neq 0 \quad \text{for all} \quad (x,t) \in \mathbf{R} \times [0;T].$$

Equation (1) arises from replacing the constant coefficients in classic KdV equation

$$u_{xxx} = u_t + 6uu_x \tag{3}$$

with variable ones.

Due to variable coefficients we expect that vcKdV equation has solutions closed to soliton solutions of equation (3). This motivates our interest in finding particular solutions of vcKdV equation 1) as asymptotic expansions in a small parameter ε.

Such solutions are called **asymptotic soliton-like solutions**.

We consider technique of constructing the asymptotic soliton-like solutions and examine the accuracy of the solutions constructed. Additionally, we address the existence of global asymptotic soliton-like solutions.

Examples illustrating the results are presented.